

EXAMEN TEMAS 1 y 2. MATEMÁTICAS I.

$$1) a) \frac{\left(\frac{3}{2}\right)^{-2} \cdot \left(\frac{4}{3}\right)^{-3}}{2^{-4} \cdot 3^{-3}} = \frac{\left(\frac{2}{3}\right)^2 \cdot \left(\frac{3}{2^2}\right)^3}{2^{-4} \cdot 3^{-3}} = \frac{2^2 \cdot 3^{-2} \cdot 3^3 \cdot 2^{-6}}{2^{-4} \cdot 3^{-3}} = 2^{2-6+4} \cdot 3^{-2+3+3} = 3^4$$

$$b) \sqrt{3\sqrt{3}\sqrt{3}} = \sqrt[4]{3^2 \cdot 3\sqrt{3}} = \sqrt[4]{3^3\sqrt{3}} = \sqrt[8]{3^6 \cdot 3} = \sqrt[8]{3^7}$$

$$c) \sqrt{x} \cdot \sqrt[3]{x} \cdot \sqrt[4]{x^3} = x^{\frac{1}{2}} \cdot x^{\frac{1}{3}} \cdot x^{\frac{3}{4}} = x^{\frac{1}{2} + \frac{1}{3} + \frac{3}{4}} = x^{\frac{6+4+9}{12}} = x^{\frac{19}{12}} = \sqrt[12]{x^{19}} = x^{12}\sqrt{x^7}$$

$$d) \frac{a}{a^4 \sqrt[6]{a^8}} = \frac{a}{a^4 \cdot a^{\frac{8}{6}}} = \frac{a}{a^4 \cdot a^{\frac{4}{3}}} = \frac{a}{a^{\frac{16}{3}}} = a^{1 - \frac{16}{3}} = a^{-\frac{13}{3}}$$

$$e) \frac{4}{5}\sqrt{8} - \sqrt{50} + \sqrt[3]{2} + \frac{7}{3}\sqrt{18} - 2\sqrt[3]{16} = \frac{8}{5}\sqrt{2} - 5\sqrt{2} + \sqrt[3]{2} + 7\sqrt{2} - 4\sqrt[3]{2} = \frac{18}{5}\sqrt{2} - 3\sqrt[3]{2}$$

$$2) a) \frac{x^3 - 5x^2 + 8x - 4}{x^3 - x^2 - 8x + 12} = \frac{(x-2)(x-2)(x-1)}{(x-2)(x-2)(x+3)} = \frac{x-1}{x+3}$$

$$\begin{array}{r|rrrr} 1 & -5 & 8 & -4 \\ 2 & & 2 & -6 & 4 \\ \hline & 1 & -3 & 2 & 0 \\ 2 & & 2 & -2 \\ \hline & 1 & -1 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 1 & -1 & -8 & 12 \\ 2 & & 2 & 2 & 12 \\ \hline & 1 & -1 & -6 & 0 \\ 2 & & 2 & 6 \\ \hline & 1 & 3 & 0 \end{array}$$

$$b) \frac{x}{x-5} - \frac{2x-1}{x+5} - \frac{50}{x^2-25} = \frac{x(x+5) - (2x-1)(x-5) - 50}{x^2-25} =$$

$$= \frac{x^2 + 5x - 2x^2 + 10x + x - 5 - 50}{x^2 - 25} = \frac{-x^2 + 16x - 55}{(x-5)(x+5)}$$

$$= \frac{(x-5)(-x+11)}{(x-5)(x+5)} = \frac{-x+11}{x+5}$$

$$\begin{array}{r|rrrr} & -1 & +16 & -55 & \\ 5 & & -5 & 55 & \\ \hline & -1 & 11 & 0 & \end{array}$$

3)

a)  $(\sqrt{2x^2+2x-3})^2 = (-1-x)^2$

$$2x^2+2x-3 = x^2 + 1 + 2x$$

$$x^2 - 4 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

comprobación

•)  $x = 2$

$$2 + \sqrt{8+4-3} = -1$$

$$2 + 3 = -1$$

X No vale

•)  $x = -2$

$$-2 + \sqrt{8-4-3} = -1$$

$$-2 + 1 = -1$$

✓ si vale

b)  $\frac{x^2-1}{x+2} \leq 0$

$$x^2-1 = (x+1)(x-1)$$

$$x+2 = 0$$

$$x = -2$$

	-2	-1	1	
$x-1$	-	-	-	+
$x+1$	-	-	+	+
$x+2$	-	+	+	+
$\frac{x^2-1}{x+2}$	-	+	-	+

Sol:  $(-\infty, -2) \cup [-1, 1]$

c)  $\log x - 2\log 3 = 1$

$$\log x - \log 9 = \log 10$$

$$\log \frac{x}{9} = \log 10$$

$$\frac{x}{9} = 10$$

$$x = 90$$

comprobación

$$\log 90 - 2\log 3 = 1$$

✓

c)  $3^{2x} - 3^{x-1} = 3^{x+1} - 1$

$$(3^x)^2 - \frac{3^x}{3} = 3 \cdot 3^x - 1$$

$$t = 3^x$$

$$t^2 - \frac{t}{3} = 3t - 1$$

$$3t^2 - t = 9t - 3$$

$$3t^2 - 10t + 3 = 0$$

$$t = \frac{10 \pm \sqrt{10^2 - 4 \cdot 3 \cdot 3}}{2 \cdot 3} = \frac{10 \pm 3}{6} = \begin{matrix} 3 \\ \frac{1}{3} \end{matrix}$$

Deshacemos el cambio:

$$t = 3^x$$

$$3 = 3^x$$

$$\Rightarrow \boxed{x=1}$$

$$\frac{1}{3} = 3^x$$

$$3^{-1} = 3^x$$

$$\boxed{x=-1}$$

4.

$$a) \begin{cases} 3x + 2y \geq 6 \\ x - y + 1 \geq 0 \end{cases}$$

$$3x + 2y = 6$$

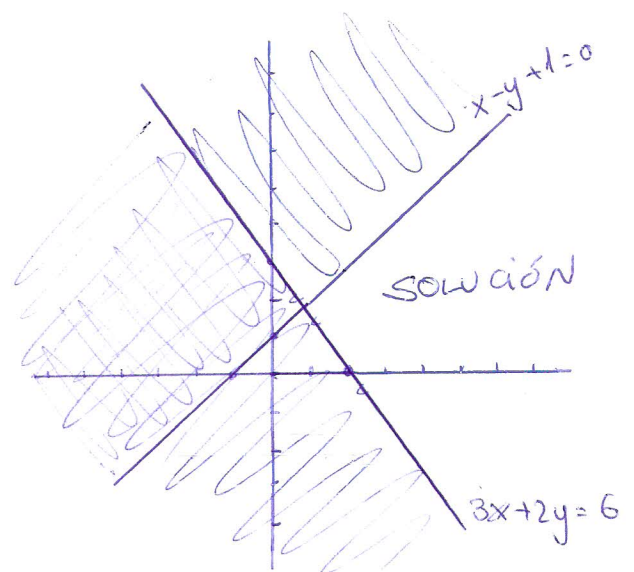
x	y
0	3
2	0

$$3 \cdot 0 + 2 \cdot 0 \geq 6 \quad \times$$

$$x - y + 1 = 0$$

x	y
0	1
-1	0

$$0 - 0 + 1 \geq 0 \quad \checkmark$$



$$b) \begin{cases} x + y = 1 \rightarrow x = 1 - y \\ xy + 2y = 2 \rightarrow (1 - y)y + 2y = 2 \end{cases}$$

$$y - y^2 + 2y = 2$$

$$-y^2 + 3y - 2 = 0$$

$$y = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot 2}}{-2} = \frac{-3 \pm 1}{-2} = \begin{matrix} 1 \\ 2 \end{matrix}$$

$$y = 1 \rightarrow x = 1 - 1 = 0$$

$$y = 2 \rightarrow x = 1 - 2 = -1$$

solución:

$$\boxed{\begin{matrix} (x=0, y=1) \\ (x=-1, y=2) \end{matrix}}$$

$$c) \left\{ \begin{array}{l} \frac{2x+3}{3} - \frac{y+1}{6} = 2 \\ \frac{5(x-1)}{2} - \frac{2y-1}{3} = 0 \end{array} \right\} \rightarrow \left\{ \begin{array}{l} 2(2x+3) - (y+1) = 2 \cdot 6 \\ 15(x-1) - 2(2y-1) = 0 \end{array} \right\} \rightarrow \left\{ \begin{array}{l} 4x+6-y-1=12 \\ 15x-15-4y+2=0 \end{array} \right\}$$

$$\left\{ \begin{array}{l} 4x-y=7 \\ 15x-4y=13 \end{array} \right\} \rightarrow \begin{array}{r} -16x+4y=-28 \\ 15x-4y=13 \\ \hline -x=-15 \\ x=15 \end{array} \quad \begin{array}{l} \Rightarrow y=4x-7 \\ y=4 \cdot 15-7=53 \end{array}$$

$$d) \left\{ \begin{array}{l} x+3=y \\ 2x=y-2 \end{array} \right\} \rightarrow y=2x+2 \quad \begin{array}{l} \Rightarrow x+3=2x+2 \\ x-2x=2-3 \\ -x=-1 \\ x=1 \\ y=4 \end{array}$$

5.

$x = n^{\circ}$  billetes de 10 €

$y = n^{\circ}$  billetes de 20 €

$z = n^{\circ}$  billetes de 50 €

$$\left\{ \begin{array}{l} x+y+z=8 \\ 10x+20y+50z=290 \\ x=2y \end{array} \right\} \rightarrow \left\{ \begin{array}{l} x+y+z=8 \\ x+2y+5z=29 \\ x-2y=0 \end{array} \right\} \rightarrow \left\{ \begin{array}{l} x+y+z=8 \\ y+4z=21 \\ -3y-z=-8 \end{array} \right\}$$

$$\rightarrow \left\{ \begin{array}{l} x+y+z=8 \\ y+4z=21 \\ 11z=55 \end{array} \right\} \rightarrow \begin{array}{l} x=8-1-5=2 \\ y=21-4 \cdot 5=1 \\ z=\frac{55}{11}=5 \end{array}$$

Solución:

2 billetes de 10 €

1 billete de 20 €

5 billetes de 50 €